AN INTERFERENCE METHOD FOR THE MEASUREMENT OF THE SPEED OF SOUND IN LIQUIDS.

BY

LLOYD BALDERSTON, JR.

Thesis presented to the Faculty of Philosophy of the University of Pennsylvania in partial fulfillment of the requirements for the degree of Ph. D.

Randal Morgan Laboratory of Physics.

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THE speed of propagation of sound waves in air has been extensively investigated both mathematically and experimentally. Newton (1) showed that in an elastic fluid pulses are propagated with a speed proportional directly to the square root of the elasticity (vis elastica) and inversely to the square root of the density. He deduces for the speed under standard conditions the value 979 feet per second, which is much too low, and the hypothesis which he proposed as an explanation of the discrepancy was soon shown to be untenable.

This problem, finally solved by Laplace, (2) furnishes one of the most interesting chapters in the history of physical science. Laplace showed that the heat developed by compression was the cause of the discrepancy between Newton's value and the true one. He proved that $\frac{E\phi}{E\theta} = \frac{Cp}{Cv}$ where $E\phi$ is the adiabatic and $E\theta$ the isothermal elasticity, Cp the specific heat at constant pressure and Cv that at constant volume. The ratio of the specific heats is practically constant for diatomic gases, and is commonly denoted by K. Laplace's correction factor is therefore $\sqrt{K} = \sqrt{1.41} = 1.19$ nearly.

The academicians of Florence in the 17th century, having experimented with water enclosed in silver spheres, concluded that it was incompressible. Hence it was concluded that water was not elastic and could not transmit sound waves. John Canton, however, showed before the Royal Society in 1762 that water can be compressed, and Franklin afterward experimented on the transmission of sound through water. The field thus opened has been much less fully investigated than that in which gases are involved.

When Laplace had supplied the correction to Newton's formula, the question arose whether or not any such factor must be

(1) Principia, Book II, Prop. 48, 49, 50.
(2) Mecanique Celeste, Vol. V, Book XII, Ch. 3, Sect I.

used in the case of liquids. This problem was attacked in a number of ways by different investigators. Colladon and Sturm (1) of Geneva made direct experiments on the development of heat by compression. In the case of ether, a blow producing compression equal to that due to a pressure of 40 atmospheres caused a rise of 6 degrees C. in temperature. A like blow in the case of water produced no perceptible effect. They then undertook to determine the speed of sound in free water, in order to compare the actual speed with that deduced from the formula $v=\sqrt{\frac{E_\theta}{\rho}}$ where E_θ is the isothermal elasticity and ρ the density. The two results agreed very closely. They therefore concluded that in the propagation of sound waves in water, thermal effects due to the disturbance do not perceptibly influence the speed.

The measurements were made on Lake Geneva in the autumn of 1826. The two stations selected were 13487 meters apart, (with a possible error of 20 meters). A bell 70 cm. high and slightly less in diameter, suspended in the water at the depth of a meter, was struck by a hammer. A torch was so connected to the lever carrying the hammer that it fired a charge of powder at the same instant that the bell was struck. The observer at the other station was provided with a quarter-second stop-watch and a trumpet, suitably placed to catch the sound from the bell. The experiments were conducted at night. The mean temperature of the water was 8.1 degrees and the average depth between the stations 140 meters. The water contained 167 parts per million total solids and .034 volume of air.

Forty-four observations of the time between the flash and the sound gave results varying from 9 to 9½ seconds, with a mean of 9.4. This gives as the mean value of the speed, 1435 meters per second. The minimum value, obtained by using the greatest observed time and least possible distance, was 1417 meters; and the maximum, obtained in a like manner, 1500 meters per second. The extreme variation thus amounts to nearly 6 per cent. of the total. The best data available at that time for the compressibility of water gave for the calculated value 1428 meters. More recent data increase this to 1437, showing a concordance which is really remarkable. The authors refer in their memoir to a measurement made

(1) Annales de Chemie et de Physique, Series 2, Vol. 36. (1827).

"a few years" before by M. Beudant in sea-water at Marseilles as the only experiment which had previously been tried. The method used was not very exact. Beudant's result was 1500 meters.

A long series of experiments on the speed of sound was conducted by M. G. Wertheim (1) at Paris. His method was indirect He immersed organ pipes in water and sounded them by forcing a current of water through them. The note was determined by comparison with a sonometer. Varying the pressure under which the water was forced through the pipes caused them to give different harmonics. From these the pitch of the fundamental was calculated. Certain corrections depending on the shape and diameter of the pipe having been applied, the speed of sound in the liquid was obtained by multiplying the wave-length by the vibration number. The mean of 58 experiments at temperatures between 15 and 20 degrees was 1173.4 meters per second.

This is much below the absolute value, if we may so call the speed in free water. Wertheim reasoned that the column of water in the tube behaved like a rod of metal. A part of the energy of the source is transmitted along the rod and a part is communicated to the surrounding medium by the lateral motion of the sides of the bar. The rod being thus free to expand laterally, a blow on the end will manifestly be propagated less rapidly than would a disturbance moving from a point in an infinite mass of the metal. Wertheim calculated that the speed in free water would be greater than in a column of water contained in a tube in the ratio of $\sqrt{3}:\sqrt{2}$. Multiplying his result by this ratio he obtained the value 1437 meters per second, precisely that given by theory from the compressibility at the temperature 8 degrees. He does not give the temperature for each experiment of the 58 but says that they were all carried out at temperatures between 15 and 20 degrees. The result 1173.4 is given in his summary as for 15 degrees.

Wertheim also experimented with various solutions in water, with alcohol, turpentine and ether, and with water at various temperatures. His values for water at 30, 40, 50 and 60 degrees are each based on five or six observations, the extremes differing in each case by more than 100 meters, with a probable error of about 1 per cent. In the table are given his values in column A, the same multi-

(1) Annales de Chemie et de Physique, Series 3, Vol. 23, (1848).

plied by $\sqrt{\frac{3}{2}}$ in column B, while in column C are the values calculated from the elasticity and density, including the correcting factor $\sqrt{\frac{Cp}{Cv}}$.

Temp.	A	B	C
30°	1250.9	1528.5	1529.5
40°	1324.8	1622.5	1664.5
50°	1349.0	1652.0	1601.3
60°	1408.2	1724.7	1622.6
70°			1639.3
80°			1650.4

The method of stationary waves devised by Kundt for measuring the speed of sound propagation in solids and gases, he has applied also to liquids. (1) He used six glass tubes, differing in internal diameter and thickness of wall. The powder used was of iron. His results were fairly concordant. Assuming that in the neighborhood of 20 degrees the speed of sound in free water increases 4 m. per second for each degree increase of temperature, and reducing Kundt's results to 20 degrees, they vary from about 1044 m. for a tube 22 mm. thick and 28 mm. in diameter to 1375 m. for one 5 mm. thick and 14 mm. in diameter. He concluded that the speed is dependent on the diameter of the tube and the thickness of the walls, quoting a passage from Helmholtz (2) in support of this view.

Threlfall and Adair (3) experimented on the speed of propagation in water of waves due to explosions, in the harbor of Port Jackson, Australia. They used charges of gun cotton of various sizes. The distance was less than 250 meters, and the measurement of time was made by a special form of chronograph. They found values varying with the quantity of explosive used, the highest being 2013 meters per second.

Professor Tito Martini (4) conducted an extensive research on this subject in 1884-5. His method was based on the sounds produced by the efflux of water from tubes, a phenomenon first de-

(1) Poggendorf's Annalen, (1874), Vol. 153, p. 1.
(2) Fortschritte der Physik, 1848, P. 114.
(3) Proc. Royal Soc. 1889, Vol. XLVI, P. 496.
(4) Atti del Reale Instituto Veneto, Series VI. Vol. IV, Appendix.

scribed by Savart (1) in a posthumous memoir presented to the Paris Academy by Arago in 1853. Martini used a tube of glass or brass, the end being closed by a brass plate 25 mm. thick with a hole 2.5 mm. in diameter in the middle. Water flowed into the tube from a reservoir. He found difficulty in regulating the inflow so as to maintain a constant level while the water was flowing out at the bottom. To obviate this, the water was conveyed by means of a rubber hose to the bottom of the tube, where it flowed in through the opening, overflowing at the top. The rate of flow was controlled by a stopcock. When the liquid column in the upright tube reached a height of from 18 to 20 cm. the note began, and fell in pitch as the column lengthened. The pitch was measured by the same method which Wertheim had used, a sonometer being tuned to unison with the note given by the water tube, and the pitch calculated from the length of the string.

Using glass tubes of various lengths but the same diameter (3 cm.), he found that the vibration period was not proportional to the length of the tube. In the case of a tube whose gravest mode of vibration gives a certain note, if we assume the length of the tube to be ¼ the wavelength of that note, the speed of sound in water will be four times the product of the vibration number by the length of the column. Proceeding in this manner Martini obtained values at 4.7 degrees varying from 1394 meters for a tube 207 mm. long to 1592 for one 546 mm. in length. The value 1435 m. identical with that found by Colladon and Sturm by direct measurement at 8.1 degrees, was given by a tube 30 cm. long. Martini therefore assumed that tubes whose length is 10 times their diameter will give by this method absolute values for the speed of sound in liquids. In his further experiments on water he used five brass tubes, each having its length and diameter in the ratio 10:1. He also worked with alcohol, ether, kerosene and several water solutions. The results for water at various temperatures are given in the table:

(1) Comptes Rendus; August, 1853.

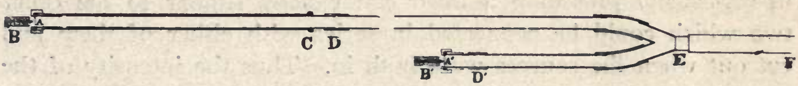
Temperature C.	Length of Column, cm.	Speed of Sound m.	Mean. Meters sec.	Probable Error, m.
	60	1353.2		
3.9°	50	1435.7	1398.6	11.3
	40	1383.8		
	30	1422.0		
	60	1374.5		
7.6°	40	1407.5	1409.0	6.7
	30	1442.0		
	20	1412.0		
	60	1399.4		
13.7°	50	1454.5	1437.3	10.1
	40	1429.0		
	30	1466 2		
	60	1432.0		
25.2°	50	1471.2	1457.2	8.0
	40	1442.2		
	30	1482.6		

An important contribution to the mathematical theory of the propagation of sound waves in liquids was made by Clausius, who showed (1) that the specific heats are connected by the relation $Cp - Cv = \frac{a^2 \ \theta \ E\theta}{p}$, where a is the coefficient of volume increase with temperature, θ is the absolute temperature, ρ the density and $E\theta$ the elasticity at constant temperature. As in the case of gases, $V = \sqrt{\frac{E\phi}{\rho}}$ and $\frac{E\phi}{E\theta} = \frac{Cp}{Cv}$ In liquids, however, $\frac{Cp}{Cv}$ is not constant. At 4° $a = o$ and $\frac{Cp}{Cv} = 1$. As the temperature rises $\frac{Cp}{Cv}$ increases. Since the work of Colladon and Sturm was done in water at 8.1 degrees, for which temperature the correction factor $\sqrt{\frac{Cp}{Cv}}$ scarcely differs from 1, it is not surprising that they found no correction factor necessary. Their determination is the only direct measurement of the speed of sound in fresh water which has found a place in our scientific literature. Their result

no doubt merits the confidence which it has received, but with our improved methods of time measurement it is certainly worth while to repeat the work. At least two determinations should be made, at temperatures as widely different as possible.

The various laboratory methods which have been employed are necessarily indirect, and in most cases the results obtained were not assumed to be absolute values. Wertheim's method has a limit of error as high as 4 per cent. Even if this could be made much less, the method would still be unreliable in so far as concerns absolute results, for Kundt's work has shown that V in liquid columns in tubes is a function of the diameter of the tube and the thickness of its wall. Wertheim's correction factor, $\sqrt{\frac{3}{2}}$ is thus proved to have no logical basis. Helmholtz showed in the article before referred to, that columns of liquid in tubes cannot behave like rods of metal, since the liquid is not entirely free to expand laterally. The diminished speed in the tube, as compared with free water, is due to the yielding of the walls of the tube, and the diminution is greater as the walls are made thinner. In so far as accuracy of measurement is concerned, Martini's method appears to possess little advantage over Wertheim's. He assumed that his results were absolute values, but the assumption seems to rest on a rather doubtful basis.

A laboratory method giving relative results only might be useful in confirming the theoretical values at higher temperatures and in investigating other liquids. The experiments here described were made in an attempt to devise a method based on interference. The plan adopted is that shown in the figure. The sources are telephone diaphragms A, A′, 5 cm. in diameter, supported between rubber gaskets and presenting a free surface 3.8 cm. in diameter. These are actuated by the electro-magnets B, B′, through which an intermittent current passes. These magnets have cores of Norway iron 6.4 mm. in diameter and 8.5 cm. long, and are wound with 720 turns each of No. 18 wire. The sliding joint C permits an extension of 80 cm. At D and D′ the brass tube, 3.8 cm. external



(Two meters of the longer tube are omitted.)

diameter and 1.6 mm. wall, is discontinuous, being joined by a piece of rubber hose for the purpose of sound-insulation. The brass Y marked E is joined to the two tubes by rubber connections, and is extended by another piece of rubber hose, F, bent up to prevent the escape of the water. The brass tubes are supported on a board 5 m. long, the parts E and F being suspended by rubber bands to a projecting arm, so as to insulate them as well as possible from the table. With the idea of preventing the transmission of sound by the walls of the tube, rubber hose was at first used from D to E. The amount of energy received at E from the more distant source was insignificant, most of it being dissipated through the yielding walls of the tube.

If now two sounds of the same pitch and in the same phase are produced at A and A′ and their intensity be so adjusted that they produce equal effects at F, there should be destructive interference when the distances of the two sources from F differ by a half wave-length. The intermittent current was at first furnished by a device similar to the commutator of an ordinary dynamo, carried on the spindle of a small motor. One brush touches a solid brass ring. The other is alternately in contact with brass and fibre. A switch placed conveniently near the observing end sent current through one magnet or the other or both in parallel. It was found that when both sources were acting there were present overtones of considerable amplitude which were not heard when one source only was in operation. This effect seems to have been due to mutual inductive action of the two sources. While the current from the battery was interrupted the electro-magnets being connected in parallel were on a closed circuit of their own. The residual magnetism present in the core during the brief interval, and the overtones of the diaphragm made of each source a telephonic transmitter, while the other acted as a receiver. The effect was thus cumulative, and the quality of the sound when both sources were acting was conspicuously different from that of the sound from a single source. This difficulty was obviated by placing the sources in series and providing a third coil exactly similar to the other two which could be connected in series with either of them and cut out when the sources were both in. Thus the intensity of the current was the same, whether one or both magnets were acting.

The necessary connections were arranged by means of a special three point switch.

The rate of interruption of the current was ascertained by means of a speed counter attached to the motor. This rings a bell at intervals of 382½ revolutions of the armature, and as the interrupting disk has 8 brass sectors, each tap of the bell corresponds to 3060 vibrations. Sparking of the interrupter was prevented by placing an adjustable condenser across the break. In order to get a reasonably pure tone, the interrupter must be accurately made and kept clean and the brushes must fit neatly on its surfaces.

It is necessary to have the apparatus completely filled with water, without air bubbles. A small bubble near one of the diaphragms interferes seriously with the intensity of the sound transmitted through the water. In order to avoid bubbles, the water was boiled and put into the apparatus while still hot, so that in cooling it might absorb any air which had adhered to the walls of the tube.

At first the apparatus was closed at E by the diaphragm of a multiple contact microphone. A reservoir communicating with the long tube by a small hole was placed near D to supply water to keep the tube full when the length was increased The microphone receiving apparatus was not satisfactory, probably on account of confused multiple reflections of overtones from the diaphragm and the walls of the tube. The form shown in the figure was afterward adopted. The reservoir was discarded and the level of the water kept nearly constant by pouring in more when the tube is longer. The adjustment of the length of the tube is made by means of a lever at the observing end, attached to a rod which in turn is secured to the carriage of the sliding portion by a clamp. Observations are made simply by placing the ear at the end of the tube F Of course reflection takes place at the air surface, but much less intense than in the case of the diaphragm, and the reflected waves are so scattered on account of the obliquity of the surface that they hinder observation but little.

The motor used is of the series type, and was driven by from 12 to 16 storage cells. The conditions were kept apparently constant, but the speed of the motor varied slightly. It seemed probable that this variation was the cause of the difficulty in obtaining sharply defined minima. A small fan with pivoted spring-held blades was put on the spindle of the motor to act as a governor, but proved inefficient.

In order to secure constancy in the pitch of the sources, a vibrating interrupter was substituted for the rotary one. A steel bar was clamped to a heavy iron base and kept in vibration in the same manner as an electrically driven tuning fork. The end of the bar carries a platinum point which dips into mercury and so interrupts the current which passes to the sources. The vibrator works very well when driven by two cells of storage battery. The current which passes to the electromagnets B and B is furnished by four storage cells. The vibrator was tuned by comparison with a Konig fork to 192 vibrations per second, in order that a resonator responding to that note might be used in listening at F. The object of this was to get rid of the overtones which helped to mask the minima.

The platinum point which dips into the mercury was attached to a slender brass spring, tuned to the same pitch as the vibrator in order to secure greater amplitude. The spring was about 35 mm. long, and its amplitude at the free end was as great as 3 mm., about 10 times that of the end of the vibrator bar to which it was screwed. Such an amplitude was not only not necessary, it was not satisfactory, drops of mercury being thrown out of the cup by the platinum tip. The brass spring was discarded and another made whose natural period was about 1½ times as great. This gave sufficient amplitude, about .6 mm., but the pitch of the vibrator rose about 12 vibrations per second when the change had been made. The first spring weighed 1 gram, the other .83 grams. In order to see whether the difference in weight could account for the change of pitch, one of the screws by which the spring was attached to the bar was taken out. Its removal changed the pitch by less than half a vibration per second. The screw weighed .27 gram. The part of the bar in vibration weighs about 90 grams. It is evident that the retarding effect of the first spring was largely due to its synchronism with the bar.

It was difficult to make this interrupter work smoothly. It has a tendency to spark explosively at unexpected moments, in spite of carefully adjusted condenser capacity. Six or eight storage cells may be be used with the rotary interrupter, giving a current of three or four amperes through the sources, but the vibrator does not work well with more than four cells. The tones obtained were not so pure as those given by the other device, but the resonator was expected to eliminate the overtones. This hope, however, proved illusory. The vibrations sent out into the air of the room by various parts of the apparatus, particularly by the longer brass tube, so affected the sensitive resonator that it was practically useless. The vibrator had been put in a closet on a pad of cotton so that it did not contribute to the diffused sound which affected the resonator. The overtones in the sound transmitted through the water made it very difficult to locate the minima at all closely. The invariability of the note also introduced a difficulty because it was hard to avoid being influenced by the previous readings. The region within which the minimum lay could be located quite easily, but over a space of twelve or fifteen centimeters the variation was very slow.

On account of the greater purity of the sounds obtained, the motor interrupter was again tried. Another motor was selected, having a better balanced armature, but it was no more uniform in speed than the other. A regulating fan, of the type already described but much larger, was connected to the motor by a belt. The driving current was then adjusted to give about 192 vibrations, so that the constancy of the pitch could be tested by comparison with the standard fork. The note was found to vary two or three vibrations per second, rising and falling from one to four times per minute.

The results so far obtained are given in the table which follows. Observations 1 to 4 were taken with the vibrating interrupter, the others with the rotary one. To make the results more easily comparable they are reduced to 20 degrees by adding or subtracting 3.2 meters for each degree below or above 20 degrees. The values found are about .8 of that for free water. The correction used is therefore .8x4 meters.

n=vibration frequency.

d=distance between sources, cm.

t=temperature, centigrade.

v=speed of propagation, meters per sec.

v_{20}=speed reduced to 20 degrees.

No.	n	d	t	v	v_{20}
1	192	313.5	20.3	1203.8	1202.8
2	192	314.	20.	1205.8	1205.8
3	192	313.	20.	1201.9	1201.9
4	192	314.	21.4	1205.8	1201.3
5	170	351.5	15.8	1195.1	1208.5
6	178	336.5	18.6	1197.9	1202.4
7	187	318.5	18.5	1191.2	1196.0
8	193.7	316.	16.3	1224.0	1235.8
9	200.5	306.5	16.3	1229.0	1240.8
10	185.2	323.	21.2	1196.4	1192.6
11	189.	319.	20.	1205.8	1205.8
12	192.	318.5	20.5	1223.0	1221.4
13	192.	317.	20.5	1217.3	1215.7
				Mean	1210.6

The uncertainty introduced by variation of the speed of the motor amounts to 1½ per cent. of the quantity measured. Two other causes contribute to make the minima difficult to fix with accuracy. Some sound is conveyed to the ear by the solid parts of the apparatus. Some confused sound is also conveyed by the water itself. This last difficulty results from the fact that every part of the tubes acts as a source of sound. When a condensation is sent out from the source, it sets up lateral vibrations in the walls of the tube. These waves travel along the tube at a rate different from that of the sound wave in the water, and the vibrations set up in the water by the walls of the tube arrive at the point of observation in all phases. Sound of the fundamental pitch due to these two causes is thus heard at F when the two main sources are in interference.

It will be seen that the method is capable of a degree of accuracy as great as that given by other indirect methods, even with the imperfect apparatus employed. By using a phonic wheel (1) or other device to give uniform speed to the interrupter, and by more perfect insulation of the apparatus, it is hoped that a much higher degree of accuracy can be reached. Another point which needs to be improved is the action of the diaphragms. Those which were used were very erratic. Both quality and loudness of the tone were subject to sudden changes without apparent cause, necessitating frequent readjustment. Special diaphragms can probably be devised which will be better.

Several possible sources of constant error have not been investigated. Perhaps the most important of these is lack of exact coincidence of phase in the sources. In so far as this might result from differences in the magnets it could be eliminated by interchanging them. But it might result also from differences in the diaphragms and in the distance between diaphragm and magnet pole. Such differences could be detected by optical methods.

(1) Lord Rayleigh's Scientific Papers, Vol. II, Pages 179-180.

It will be seen that the method is capable of a degree of precision as great as that given by other indirect methods, even with the imperfect apparatus employed. By using a phonic wheel (14) or other device to give uniform speed to the interrupter, and by more perfect insulation of the apparatus, it is hoped that a still higher degree of accuracy can be reached. Another point which seems to be important is the action of the diaphragms. Those which were used were very erratic. Both quality and loudness of the tone were subject to sudden changes without apparent cause, necessitating frequent readjustment. Special diaphragms can probably be devised which will be better.

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(14) Lord Rayleigh's Scientific Papers, Vol. II, Paper [illegible]